LAPLACIAN COMPARISON THEOREM ON RIEMANNIAN MANIFOLDS WITH $\mathrm{CD}(K,m)$-CONDITION FOR $m \leq 1$

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Abstract

Let $L = \Delta - \nabla\phi\cdot\nabla$ be a symmetric diffusion operator with an invariant measure $\mu(dx) = e^{-\phi(x)}m(dx)$ on a complete non-compact smooth Riemannian manifold (M,g) with its volume element $m = \mathrm{vol}_g$, and $\phi \in C^2(M)$ a potential function. In this talk, we show a Laplacian comparison theorem on weighted complete Riemannian manifolds with $\mathrm{CD}(K,m)$-condition for $m \leq 1$ and a continuous function K. As consequences, we give the optimal conditions on m-Bakry-Émery Ricci tensor for $m \leq 1$ such that the Bishop-Gromov volume comparison theorem, the finiteness of fundamental group, Ambrose-Myers' theorem, Myers' theorem, and the Cheeger-Gromoll type splitting theorem hold on weighted complete Riemannian manifolds. Some of these results were well-studied for m-Bakry-Émery Ricci curvature for $m \geq n$ (Qian, Lott, Li, Wei-Wylie) or $m = 1$ (Wylie, Wylie-Yeroshkin 2016). When $m < 1$, our results are new in the literature. This is joint work with Xiang-Dong Li (Chinese Academy of Science).

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